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Article: **Behavioural Analysis of Interbank Rates in Pakistan**

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Behavioural Analysis of Interbank Rates in Pakistan

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Abstract

Interbank rate is an important benchmark used to gauge the impact of monetary policy on the economic activities of a country. It serves not only as the basis of financial market operations but also acts as an effective tool of the central bank, which uses it to control the flow of funds, both in terms of domestic and foreign currencies. In the past, researchers used time series methodology to analyse the volatility of interbank rates which has a few drawbacks. This study, however, attempted to model the mean reversion and volatility of interbank rate by applying the Vasicek stochastic model using the maximum likelihood estimation (MLE) technique. In order to get precise observations, the study applied Monte Carlo simulation in R. The results suggested that the long term mean and speed of mean reversion behaved in a contrasting manner over different time horizons in the interbank market of Pakistan.

Keywords: interbank rate, maximum likelihood estimation (MLE), Monte Carlo simulation, R software package, Vasicek model
JEL: B41, C02, C22, C53, C58, E43, G21,

Introduction

The main goal of economic policies is to bring price stability so that sustainable economic growth can be ensured. Price stability cannot be achieved without stabilizing the interest rate which is an integral part of the monetary policy. The purpose of a monetary policy is to control the flow of funds using three channels, namely credit expansion, bank loans, and interest rates (Rossetti et al., 2017). Here, interest rate is a crucial element that is used to manage financial institutions, especially banks. One of the key elements of banking operations is setting the interbank rate which is determined on daily basis. It fluctuates in accordance with the monetary

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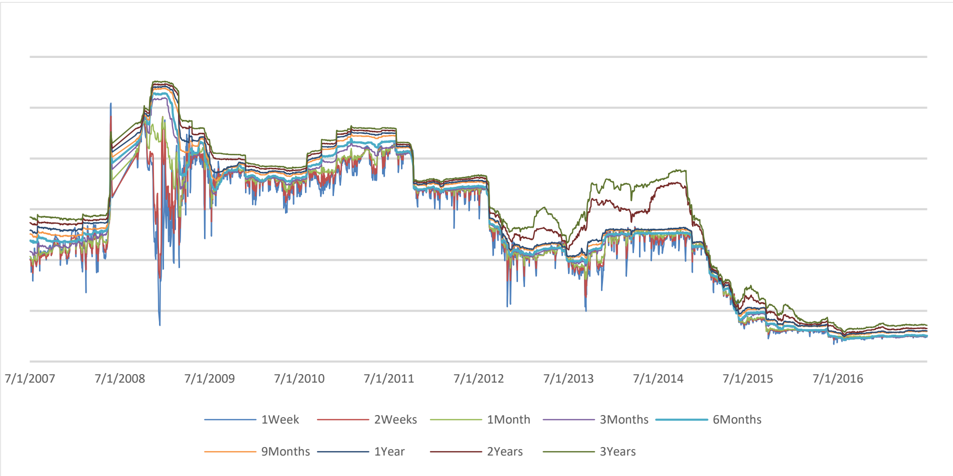
policy rate. In Pakistan, monetary policy has a direct effect on repo rate via shift in the interest rate corridor (IRC). Any shift in the interest rate corridor (IRC) affects the interbank rate as well as the overall interest rate in the country. As the value of the fixed income securities and time deposits depends on interbank rate, therefore, any volatility in the interest rate also affects the prices of long term securities, especially of government and corporate bonds. By analysing the behaviour of interbank rates, the current study attempted to understand the volatility and mean reversion characteristics of interest rates in Pakistan.

Before going into the details of the model, it is important to understand the mechanics of the interbank rate in Pakistan. Since 2004, Karachi Inter Bankoffer rate (KIBOR) is an important rate used in Pakistan as benchmark for corporate lending. Furthermore, Financial Market Association of Pakistan (FMAP) collects KIBOR rates from member banks and announces it daily at 11:30 AM on Reuters after excluding outliers such as the four extremes both on the higher and lower sides of the contributed rates by member banks. To authenticate the rate, the member banks must accept a bid/offer up to Rs.100 million from 11:30 AM to 11:45 AM at the specified rate. As per the guidelines of FMAP, the current bid/offer should not deviate from the previous rate by a certain percentage. This makes KIBOR a good example of Markov process as well. Figure 1 presents the trend line of various KIBOR rates from 2007 to 2017. Overall, in the graph, the movement of long term rates is concurrent with the short term rates, except during two segments of time. First, it was asynchronous during 2008 to 2009 and later, it was asynchronous around 2013 to 2014. The former seems to be the impact of global financial crisis (GFC) where long term rates were raised to high limits, resulting in volatility in short term rates. the latter time period was a period of political crisis in Pakistan newly elected government was facing charges of electoral fraud in the Supreme Court of Pakistan. As depicted in the below given figure, both long term and short term interest rates move in tandem with long term rates which are in line with the expectation theory of yield curve.

It is important to note that interest rates with different maturities respond differently towards changes in the policy rate. It creates uncertainty in the forecasted prices of the fixed income securities, especially the bonds. In a perfect world, the value of bond can be calculated easily because the interest rate would be fully predictable. However, in reality, bond prices and the interest rates behave in a stochastic manner. Therefore, interest rates must be modelled in a stochastic manner. Past researchers have attempted to model macroeconomic variables using the traditional time series methodology. The current study argues that the traditional time series approaches such as Auto-regressive models (ARCH, GARCH, and EGARCH) have certain limitations and cannot fully capture the stochastic nature of interest rates. The superiority of stochastic modelling over third generation GARCH models is still a debateable issue. According to Zhang (2012), stochastic models, such as the Vasicek model, capture both the mean reversion as well as the volatility based on the Weiner process. Therefore, the current study attempted to model the behaviour of interbank rates using stochastic modelling approach.

Figure 1

KIBOR Trendline



The remainder of the paper is organized as follows: section 2 deals with the review of the literature, section 3 provids the methodology, section 4 discusses the estimated results, and section 5 presents the conclusion and offers recommendations for future research. Finally, complete R codes have been presented at the end of the paper.

Literature Review

The foundation of the Vasicek model lies in the Uhlenbeck-Ornstein process, it is commonly referred as OU process and is named after the seminal work of Leonard Ornstein and George Eugene (Uhlenbeck & Ornstein, 1930). Several financial researchers have acknowledged the OU process as a fundamental concept of interest rate modelling. OU is a diffusion process based on Brownian motion. During the late 90s, researchers started using the OU process to model the volatility of asset prices. Existing literature reveals that the Vasicek model is the first single factor model that is utilized for the interest rate modelling. In his seminal work, Vasicek (1977) derived a general form of term structure of interest rate assuming diffusion property in instantaneous spot interest rate, dependence of price of discounted bond on spot rate and efficiency of market.

Although the basic assumption of economic theories/models related to the nominal interest rate is mean reversion, their empirical evidence is inconclusive. Existing literature suggests that the unit root hypothesis with respect to long term bond yields cannot be rejected using conventional unit root tests such as Dickey Fuller (Rose, 1998; Stock & Watson, 1988; Campbell & Shiller, 1991; Siklos & Wohar, 1997). Several researchers have proposed interest rate models (Merton, 1973; Richard, 1979; Cox et al., 1985; Hull & White, 1990; Pearson & Sun, 1994). However, the consistency of model parameters based on different estimation methods remained a bone of contention among researchers. Chan et al. (1992) used Generalized Method of Moment (GMM) for estimation, while Kladívko (2007) used Maximum Likelihood Estimation (MLE), and De Munnik & Schotman (1994) applied Ordinary Least Square (OLS) method of estimation for interest rate modelling. It is important to note that no one has to adopt the simple and accurate process while selecting the estimation method since simple models are less accurate and vice versa. Brown and Dybvig (1986) estimated the parameters of the Cox model using cross sectional data and found that the long term mean and volatility parameters are unstable and showed misspecification in the CIR model Cox, et al. (1985). Gibbons & Ramaswamy (1993) repudiated the results of CIR model while estimating short term T-Bills data through GMM estimation, whereas, Pearson and Sun (1994) acquired the same results by using MLE method.

The significance of mean reversion versus volatility has also been discussed in existing literature. For example, Chan et al. (1992) argued that it is not important to model mean reversion but it is necessary to model the volatility of interest rates in the correct way. The models that best describe the short term rate dynamics allow volatility to depend on the level of interest rate. Chan et al. (1992) used GMM and assumed that the distribution is ergodic. However, Nowman (1997) reported opposite results regarding the importance of mean reversion versus volatility in UK. Contrarily, Dahlquist (1996) found positive relationship between the interest rate and volatility which indicates the evidence of mean reversion in Swedish and Danish markets. Furthermore, Stanton (1997) reported non linearity in the drift coefficient in the single factor model. Other group of researchers used two factor models; for instance, (Stambaugh, 1988; Longstaff & Schwarts, 1992; Litterman et al., 1991) suggested the use of two factor model for a better fit, while Carverhill and Pang (1995) argued that the one factor Markov model provides more reliable results in the case of pricing and hedging.

Significant literature has been devoted to modelling the behaviour of spot rates in mature markets, particularly in US markets. These studies, among many others, include (Durham, 2003; Durham & Gallant, 2002; Ang & Bekaert, 2002; Elerian et al., 2001; Das, 2002; Jones, 2003; Johannes, 2004; Hong et al., 2007). However, even after extensive existing literature on the topic, evidence of emerging markets is still very limited. In a study of the South African financial market, Wet (2006) estimated the volatility of exchange rate, stock index, and interest rate by applying GARCH estimation technique and found that interest rates affect the stock index negatively. Similarly, Andritzky et al. (2007) conducted a cross country study on twelve emerging economies. They applied GARCH model to estimate the relationship between interest rate and macroeconomic variables, and found that the announcements about bond rating and US interest rates have a higher impact on volatility in comparison to the announcements of local policies. (Burgess (2014; Buzzacchi & Forster, 2016; Coskun et al., 2017) reviewed the Vasicek model and discussed its drawbacks. They reported that Vasicek short rates can be negative for certain combinations of model parameters which is logically possible, however, intuitively interest rates should not be negative. Last but not least, Orlando et al. (2020) compared the Vasicek and CIR models using the partition approach based on rolling windows from the observed financial data. They argued that the new partition approach has the ability to overcome skewed tail and volatility clustering problems.

The review of relevant literature revealed that most of the studies are either conducted on developed economies or have used traditional time series models for the estimation of interest rate behaviour. There is still a gap in understanding the dynamics of interest rate models, especially in the case of emerging economies such as Pakistan. Thus, the current study attempted to model the mean reversion and volatility of interbank rates in Pakistan using a stochastic modelling approach. The complete methodology of estimation has been discussed in the next section.

Research Methodology

Interest rate calibration is among the most challenging topics of financial econometrics (Csajková, 2005). Although the change in interest rates is a random phenomenon and is subject to the demand and supply forces of the market; however, it cannot be explained using a random walk hypothesis. Since the last two decades, extensive work on theoretical and empirical research asserts that the interest rates have mean reversion property. Initiating from Black-Scholes (2019) model to date and from single factor model to multi-factor models, mean reversion is considered a fundamental property in interest rate modelling (Black & Scholes, 2019). Existing econometric studies have focused on estimation rather than calibration. In estimation, researcher takes the observable data and tries to estimate the model parameters of a so-called 'data generating process' through which the observed data is expected to be reproduced. However, according to Cooley (1997) *"Calibration…. views the appropriate data or measurements as something to be determined in part by the features of the theory. Some of the parameter values are chosen based on observed features of actual economies, as in the traditional methods, but the determination of others may be based heavily on the theory. In this process, calibration and estimation are complements, not substitutes."* From an estimation point of view, calibration and estimation are competing methodologies used in financial economics (Hansen & Heckman, 1996; Sims, 1996; Kydland & Prescot, 1996; Cooley, 1997). Interest rate calibration chooses model parameters based on the examination of short term rates and then plugs them in simulation model to mimic and reproduced future values. Considering the Vasicek model, the current study attempted to calibrate short term KIBOR and explained the behaviour of the short term interbank market in Pakistan (Vasicek, 1977). Additionally, as per the existing literature, the current study has not developed any hypothesis because the main objective was to calibrate the model parameters rather than inferring the population parameters. The complete R codes used in calibration are given at the end of the paper.

Primarily, the current study is based on the argument of no riskless interest arbitrage which is in line with the expectation theory of the term structure of interest rates. It used the Vasicek model to model the interest rates. In order to understand the Vasicek model some important notations must be understood first. The complete derivation of the model is already published in (Vasicek, 1977; Cerny, 2012). Moreover, the explanation is kept simple to make it understandable for non-technical readers. The first step is to understand AR(1) stochastic process, with a drift such as:

$$X_t = \alpha + \beta X_{t-1} + \epsilon_t \quad ; \quad \epsilon_t \sim N(0, \theta^2) \ldots\ldots \tag{1}$$

Where X_t is a time series, α is an intercept, and β is an autoregressive coefficient. The residual term is represented by ϵ_t having a distribution normal with mean 0 and variance θ^2. If the process is mean reverting, then it will tend/bend towards long term mean β, which is the speed of mean reversion in equation 1. Typically, mean reversion in AR (1) process is measured by (α/(1-β)). In the case of interest rate, it is irrational to assume a random walk such as interest rate can never move infinitely upward or downward. On the contrary, interest rates always have mean reversion property, which means that if interest rates are high they are expected to come down and vice versa. The Vasicek model assumes the mean reversion property and also assumes that interest rates do not go below zero and revolves around long term mean. There are two important components of the Vasicek model such as the long term mean and randomness.

$$Current\ \Delta\ in\ interest\ rate = f\ [\ long\ term\ mean\ (or\ drift), random\ volitility\ of\ rates\ (or\ diffusion)]$$

Notated by Uhlenbeck- Ornstein (1930) as

$$dX_t = -\theta_2\, X_t dt + \theta_3 dW_t \tag{2}$$

Modified by Vasicek as

$$dX_t = (\theta_1 - \theta_2\, X_t)dt + \theta_3 dW_t \tag{3}$$

Generally, it is written as

$$dX_t = \theta(\mu - X_t)dt + \sigma dW_t \tag{4}$$

Where dX_t represents the current change in the interest rate, θ represents the speed of reversion, μ is the long-term mean, dt represents the change in time, σ represents the volatility of change in interest rate measured by the standard deviation of change in the interest rates, and dW_t is the Weiner process (or Brownian motion term). The state variable is X indexed by time t, while collectively, the term $\theta(\mu - X_t)$ represents drift and σ represents diffusion and diffusion. Property is represented as dW_t. In equation 4, if X_t becomes greater than μ, then the drift term becomes negative and pulls the interest downwards. The opposite would happen if $X_t < \mu$ (Zeytun & Gupta, 2007). This mean reversion property supports the economic argument that borrowers are reluctant to borrow funds when interest rates are extremely high. For this reason, economic activities tend to be sluggish and pull back interest rates to an equilibrium state. Conversely, borrowers are attracted to low interest rates and acquire more funds from markets that boost economic activities. In such cases, interest rate rises. In the absence of this economic phenomenon, the interest rate tends to drift up or down permanently which is not intuitive.

The second part of equation 4 (i.e.σdW_t) measures the instantaneous change in the interest rate (i.e. diffusion process). It is represented by Wiener[1] process W_t which contains the properties of Markov process. It

[1]Standard Wiener process $w(t), t > 0$ is a stochastic process where every increment $w(t + \Delta) - w(t)$ is normally distributed with mean 0 and variance $\Delta > 0$, and for every $0 < t1 <$

describes the probabilistic evolution of a variable over a given period of time. Being characterized by Markov property, W_t is based only on the current value without considering the historical values of the variable. In this context, KIBOR also possesses the Markov property. The Vasicek model assumes that the movement in interest rates are due to random market shocks. In the absence of market shocks when $dW_t = 0$, interest rate would be constant, that is, $r_t = \mu$. The dW_t represents the Weiner process with standard properties. The current study attempted to estimate the Vasicek model parameters and analyse the mean reversion property and long-term mean of different KIBOR rates. This study also tried to identify the reasons behind its behaviour.

To get the initial idea about the model parameter, KIBOR daily data of week 1, week 2, 1 month, 3 months, 6 months, 9 months, and 1 year was collected from 1st July 2007 to 30th June 2017. Our main objective was to calibrate the Vasicek model parameters and then simulate future interest rates. As discussed in the introduction section, this study used the first generation model. The primary reason for adopting this technique is its simplicity and tractability in pricing. As calibration of KIBOR rate could not widely attract attention of financial researchers in Pakistan therefore, present study obviously starts with simplest form of model. Initially, the model parameters were calibrated using the historical data. The method of maximum likelihood was applied to obtain the values of $\theta, \mu, and\ \sigma$ of Vasicek model. The steps taken to calibrate the model are explained below.

The first step was to model an $n \times m$ matrix of short rates for which customized R codes were used. The second step was to model the zero coupon instrument price for producing a yield curve from the Vasicek model with maturities ranging from 1 year to 10 years. In the third step, the Vasicek model parameter was calibrated using maximum likelihood estimation. Using the calibrated model parameters, required number of future trajectories based on Monte Carlo Simulation technique were obtained. Finally, the yield curve was calculated based on the given short

$\cdots < tn$ increments $w(t2) - w(t1) \ldots w(tn-1)$ are independent random variables, and $w(0) = 0$ and the sample paths of $w(t)$ are continuous. (Csajkova, 2005) rates. The graphs of simulations and yield curves are provided in appendix A.

Results and Discussion

As discussed in the previous section, the model parameters were measured using the maximum likelihood estimation. There are three main parameters of the Vasicek model, namely, θ that measures the speed of mean reversion. The lower the speed of mean reversion in the interest rate, the more it is expected to remain closer to its current value in the next period. The slow speed of mean reversion also means that the interest rate will not show a marked change, which ultimately results in parallel movement of long term bond price to/and the short term bond price. In the premise of expectation theory, high speed of mean reversion results in a steeper yield curve because the term structure depends on future expected short rates.

Table 1

Estimated Values of Model Parameters

	1 Week	2Weeks	1Month	3Months	6Months	9Months	1Year
μ	0.10052	0.10284	0.10952	0.116840	0.120433	0.123536	0.125351
θ	2.71884	1.13738	0.39085	0.234453	0.222105	0.212389	0.207757
σ	0.05576	0.035019	0.01840	0.012319	0.011607	0.011251	0.0110551
N	2514	2514	2514	2514	2514	2514	2514

Table 1 presents the values of model parameters for various short term rates based on maximum likelihood estimation in R. In the first row, μ represents the long term mean of various short term rates. It is obvious that in a normal yield curve, the long-term mean of short maturity rates is less than longer maturity rates. The next row represents the speed of mean reversion θ. It is higher in 1 week rate and gradually decreases when the term to maturity increases. As stated previously, high mean reversion means that the rate tends to revert to its long term mean, which means there is high volatility in the rate. $\boldsymbol{\sigma}$ shows the volatility which also higher when speed of mean reversion is high and vice versa. The results suggest that the long-term mean and speed of mean reversion behaves in a contrasting manner over different time horizons in the interbank market of Pakistan. 1 week rate has the lowest estimated long term mean (0.10), the highest mean reversion value of **θ** (2.72), and the highest volatility **σ** (0.06). This shows that KIBOR short rates are more volatile than the long rates. This finding is also observable in Figure 1 where 1 week rates seems to have high volatility as compared to 3 years rates. This study used historical data to estimate the parameters of the Vasicek model using MLE. Then, the estimated parameters were used to simulate future trajectories of short rates presented in Appendix A.

Conclusion

This paper attempted to estimate the Vasicek model parameters of KIBOR. It aimed to analyse the behaviour of interbank rates in Pakistan. The results show that shorter the time to maturity, lower will be the long-term equilibrium rate with high mean reversion. Furthermore, shorter maturity rates are more volatile than longer maturity rates. It was also observed that the speed of mean reversion and volatility are directly related to each other. may be due to borrowers preference towards short-term funds which are more certain rather long-term leading towards high volatility in short term rates as compare to long-term rates. This typically stands true in the context of Pakistan's political and economic conditions since its financial institutions are not interested in the auction of long-term financial instruments such as Pakistan Investment Bonds issued by the State Bank of Pakistan. Future researchers can extend this study by using the two factor model with a different estimation technique such as those discussed in the literature review. Additionally, the estimation for interest rate modelling can be improved in light of the findings given by Orlando et al. (2020), who have suggested the use of partition approach for the analysis of interest rates.

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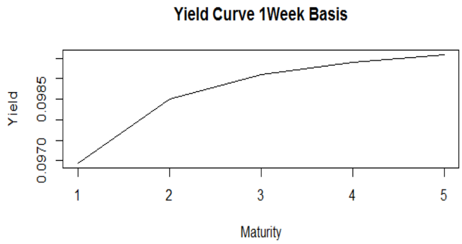
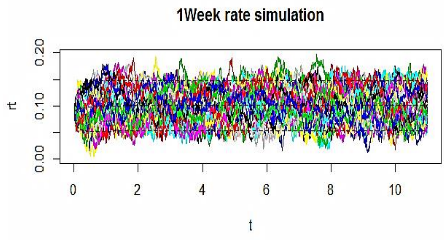
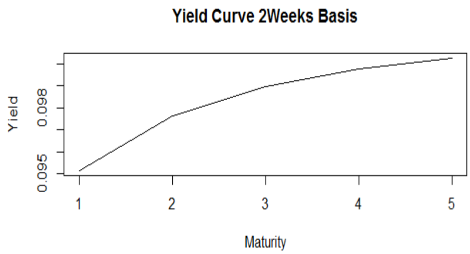
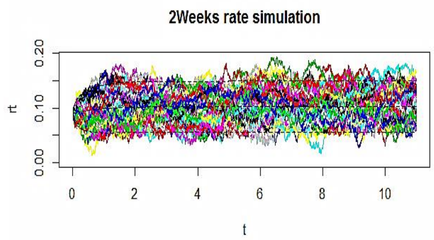
Appendix A

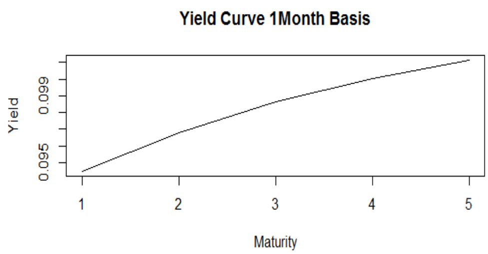
Implied yield curves (left side) and Monte Carlo simulation (right side) based on calibration for KIBOR rates of different durations.

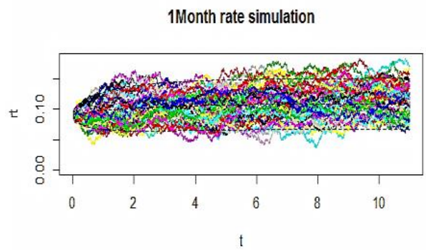


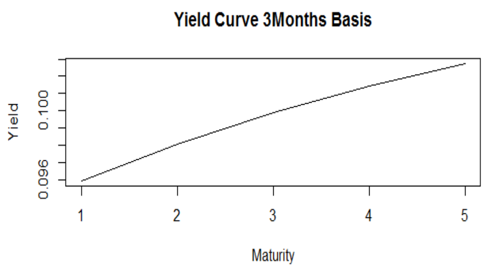


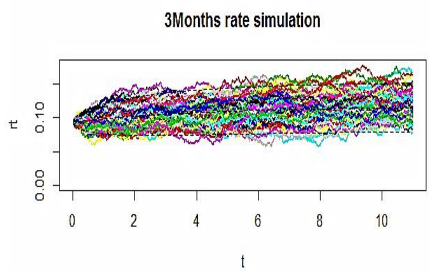


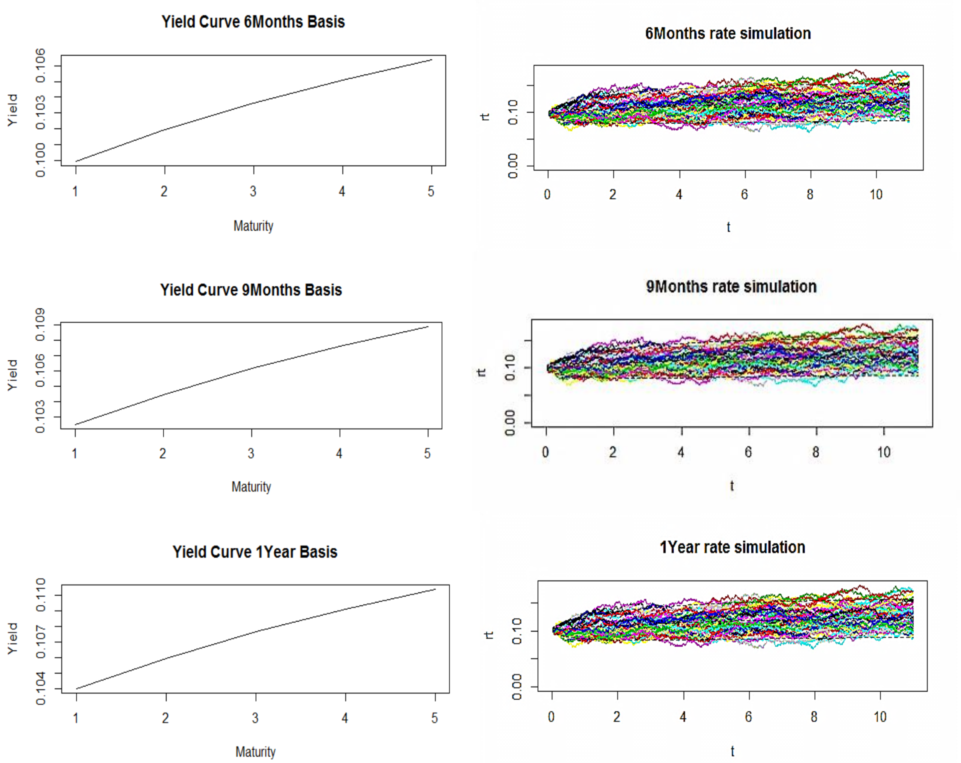












R Codes Used for Model Estimation

In D1$X1Week, D1 is the data file name and 1Week is the series title.

#B1-Helper function that calculates the next rate based on the discretization of the Vasicek model.

```
VasicekHelper <- function(r, theta, mu, sigma, dt = 1/252) {
  term1 <- exp(-1 * theta * dt)
  term2 <- (sigma^2) * (1 - term1^2) / (2*theta)
  result <- r*term1 + mu*(1-term1) + sqrt(term2)*rnorm(n=1)
  return(result)
}
```

#B2-Generates a single short rate simulation using the Vasicek model.

```
VasicekSimulation <- function(N, r0, theta, mu, sigma, dt = 1/252) {
  short.rates <- rep(0, N)
```

```
 short.rates[1] <- r0
 for (i in 2:N) {
   short.rates[i] <- VasicekHelper(short.rates[i - 1], theta, mu, sigma, dt)
  }
 return(short.rates)
}
```

#B3-Generates several short rate simulations using the Vasicek model.

```
VasicekSimulations <- function(M, N, r0, theta, mu, sigma, dt = 1/252) {
 sim.mat <- matrix(nrow = N, ncol = M)
 for (i in 1:M) {
   sim.mat[, i] <- VasicekSimulation(N, r0, theta, mu, sigma, dt)
  }
 return(sim.mat)
}
```

#B4-Calculates the zero coupon instrument price.

```
VasicekZeroIP <- function(r0, theta, mu, sigma, years) {
 b.vas <- (1-exp(-years*theta)) / theta
 a.vas <- (mu-sigma^2/(2*theta^2))*(years-
b.vas)+(sigma^2)/(4*theta)*b.vas^2
 return(exp(-a.vas-b.vas*r0))
}
```

#B5-Produces a yield curve from the Vasicek model with maturities ranging from 1 year to max.maturity.

```
VasicekYieldCurve <- function(r0, theta, mu, sigma, max.maturity=10) {
  yields <- rep(0, max.maturity)
 for (y in 1:max.maturity) {
   yields[y] <- -log(VasicekZeroIP(r0, theta, mu, sigma, y))/y
  }
 return(yields)
}
dt<-1/252
```

#B6-Calibrates the vasicek model using the maximum likelihood estimator.

```
VasicekCalibration <- function(dt = 1/252) {
 require(quantmod)
 n <- length(D1$X1Week)
```

#B7-Do the calculations
```
  Sx <- sum(D1$X1Week[1:(length(D1$X1Week) - 1)])
  Sy <- sum(D1$X1Week[2:length(D1$X1Week)])
  Sxx <- as.numeric(crossprod(D1$X1Week[1:(length(D1$X1Week) - 1)],
D1$X1Week[1:(length(D1$X1Week) - 1)]))
  Sxy <- as.numeric(crossprod(D1$X1Week[1:(length(D1$X1Week) - 1)],
D1$X1Week[2:length(D1$X1Week)]))
  Syy <- as.numeric(crossprod(D1$X1Week[2:length(D1$X1Week)],
D1$X1Week[2:length(D1$X1Week)]))
  mu  <- (Sy * Sxx - Sx * Sxy) / (n* (Sxx - Sxy) - (Sx^2 - Sx*Sy) )
  theta <- -log((Sxy - mu * Sx - mu * Sy + n * mu^2) /   (Sxx - 2 * mu * Sx
+ n * mu^2)) / dt
  a <- exp(-theta*dt)
  sigmah2 <- (Syy - 2 * a * Sxy + a^2 * Sxx - 2 * mu * (1-a) * (Sy - a *
Sx) + n * mu^2 * (1 - a)^2)/n
  sigma <- sqrt(sigmah2 * 2 * theta / (1 - a^2))
  r0 <- D1$X1Week[length(D1$X1Week)]
  return(c(theta, mu, sigma, r0))
}
```
#B8-Define model parameters and calibrate
```
years <- 11
N <- years * 252 # each year consists of 252 days
t <- (1:N)/252 # for plotting purposes
```
#B9-Calibrate the model
```
calibration <- VasicekCalibration()
theta <- calibration[1]
mu <- calibration[2]
sigma <- calibration[3]
r0 <- calibration[4]
set.seed(1)
test <- VasicekSimulation(N, r0, theta, mu, sigma)
plot(t, test, type = 'l')
```
#B10-Test with several (M = 20) simulations
```
M <- 20
test.mat <- VasicekSimulations(M, N, r0, theta, mu, sigma)
```
#B11-Plot the paths

```
plot(t, test.mat[, 1], type = 'l', main = '1Week rate simulation', ylab = 'rt',
    ylim = c(0, max(test.mat)), col = 1)
for (count in 2:ncol(test.mat)) {
  lines(t, test.mat[, count], col = count)
}
```

#B12 (Optional) -Plot the expected rate and +- 2 standard deviations (theoretical)

```
expected <- mu + (r0 - mu)*exp(-theta*t)
stdev <- sqrt( sigma^2 / (2*theta)*(1 - exp(-2*theta*t)))
lines(t, expected, lty=2)
lines(t, expected + 2*stdev, lty=2)
lines(t, expected - 2*stdev, lty=2)
```

#B13-Derive a yield curve (can do this for several values of r0 to get several curves)

```
yields <- VasicekYieldCurve(r0, theta, mu, sigma, 10)
plot(1:10, yields, xlab = 'Maturity', type = 'l', ylab = 'Yield', main = 'Yield
Curve 1Week Basis')
```