Journal of Quantitative Methods (JQM)
Volume 8 Issue 1, Spring 2024
$ISSN_{(P)}$: 2522-2252, $ISSN_{(E)}$: 2522-2260
Homepage: https://ojs.umt.edu.pk/index.php/jqm

Article QR

Title:	**Performance of PC Modified PC Algorithms of Graph Theoretic Approach: A Monte Carlo Simulation Study**
Author (s):	Rizwan Fazal[1], Atiq ur Rehman[2], Faizan Hamayat[3], Tariq Majeed[4], and Salita[5]
Affiliation (s):	[1]Sustainable Environmental and Energy Solutions, Lahore, Pakistan [2]University of Azad Jammu and Kashmir, Muzaffarabad, Pakistan [3]Shenzhen AiMall Technology Co. Ltd. Shenzhen, Guangdong, China [4]Abdul Wali Khan University, Mardan, Pakistan [5]University of Saskatchewan, Saskatoon, Canada
DOI:	https://doi.org/10.29145/jqm.81.01
History:	Received: December 28, 2019, Revised: October 21, 2021, Accepted: January 15, 2024, Published: June 29, 2024
Citation:	Fazal, R., Rehman, A., Hamayat, F., Majeed, T., & Salita. (2024). Performance of PC and modified PC algorithms of graph theoretic approach: A Monte Carlo simulation study. *Journal of Quantitative Methods, 8*(1), 1–24. https://doi.org/10.29145/jqm.81.01
Copyright:	© The Authors
Licensing:	This article is open access and is distributed under the terms of Creative Commons Attribution 4.0 International License
Conflict of Interest:	Author(s) declared no conflict of interest

A publication of
Department Of Economics and Quantitative Methods, Dr. Hasan Murad School of Management, University of Management and Technology, Lahore, Pakistan

Performance of PC and Modified PC Algorithms of Graph Theoretic Approach: A Monte Carlo Simulation Study

Rizwan Fazal[1*], Atiq ur Rehman[2], Faizan Hamayat[3], Tariq Majeed[4], Salita[5]

[1]Sustainable Environmental and Energy Solutions, Lahore, Pakistan
[2]Kashmir Institute of Economics, University of Azad Jammu and Kashmir, Pakistan
[3]Shenzhen AiMall Technology Co. Ltd. Shenzhen, Guangdong, China
[4]Department of Economics, Abdul Wali Khan University, Mardan, Pakistan
[5]Johnson Shoyama Graduate School of Public Policy, University of Saskatchewan, Saskatoon, Canada

Abstract

Keeping in view the work of Swanson and Granger (1997) among others, the performance of PC algorithm and Modified PC algorithm of graph theoretic approach in term of size and power properties are evaluated using Monte Carlo simulation. The study recommends modified PC algorithm as the dominant approach to causality as it successfully expose the correct causal relationship between variables and best to differentiate between correct and spurious causality.

Keyword**:** econometrics, time series, causality, PC and modified PC, simulations

Introduction

Causality is at the heart of social sciences, yet there is no satisfactory way to determine it. In the natural sciences, causality can be determined through controlled experiment but in social sciences control experiments are often impossible, therefore, one has to deal with observational data for the causal analysis (Lin, 2008). Several testing procedures are developed over a period of time for testing causality in observational data i.e., Granger causality (1969), Sim's causality (1972), Graph Theoretic Approach (GTA) developed by Pearl (1993), Spirtes et al. (2000). But there are serious theoretical and empirical weaknesses attached to these causality tests. After development of Granger causality, it was thought, initially, that the issue of determining the causal relation would be resolved, but it, too, has major flaws, as Granger causality determines predictability, not the causality; sometime the cause occurs later than the consequences (Fazal et al., 2022a).

*Corresponding Author: rizfaz167@gmail.com

Many authors have given elegant examples that ordering does not necessarily imply causality i.e. Hicks (1980) and many others deny to accept Granger presented causality and gave different examples in which effect occur before cause i.e., Christmas causes prices of toys to rise so cause is before the effect. Secondly, it does not take into account the temporal ordering. This approach check causality between two variables while ignore the third one. Demiralp et al. (2008) mentioned in his article that the concept of Granger causality fails to capture structural causality.

Suppose one finds that a variable *A* Granger-cause another variable *B*. This does not necessarily imply that economic mechanism exists by which *A* can be manipulated to affect *B*. This existence of such a mechanism in turn does not necessarily imply Granger causality either. (for more detail see Hoover 2001, pp. 150-155)

Other than Granger causality test, causal analysis using graphical models such as PC algorithms of graph theoretic approach are progressively applied in a variety of social sciences, but are unfamiliar to most of economists (Demiralp & Hoover, 2003). In Graph Theoretic approach, structural model is converted into graph which overcome many problems and bring causality back into the front of researcher and philosophers.

Graph theoretic methods were generally not conceived with time series data. Swanson and Granger's (1997) for the first-time used Graph theoretic approach to causality into the analysis of contemporaneous causal order of SVAR. They assume that information about causal ordering of contemporaneous variables of SVAR is actually contained in the covariance matrix of VAR error terms (Fazal et al., 2022b). Demiralp et al. (2008), Hoover (2005) showed that after estimating VAR model the error terms of that model would be stored and then treated as the original time series variables in PC algorithms to find the causal direction. Recently economists have applied GTA in different areas of economics. Theoretically PC algorithm of graphical models looks sound and can be held as a preferred approach for testing causality. But how it performs empirically? The literature carries no answer to this question. As it is not known, to what extent the PC algorithm is capable to differentiate between genuine and spurious causal assumption. So current study has evaluated the performance of PC algorithm using Monte Carlo simulations which is the first contribution of the study. The study also modified the existing PC algorithm of graph theoretic approach by replacing VAR residuals with Rehman and

Malik (2014) – Modified R recursive residuals (called Modified PC algorithm) because VAR model residuals carry only contemporaneous information about cross variable effect. Consider a VAR model:

$$y_t = \alpha_1 + \beta_1 x_{t-1} + \beta_2 y_{t-1} + \varepsilon_{1t} \dots (a)$$

$$x_t = \alpha_2 + \beta_3 y_{t-1} + \beta_4 x_{t-1} + \varepsilon_{2t} \dots (b)$$

After estimating the VAR model, extract residuals series of both equations (a) and (b). However, residual series extracted from equation (a) only effect of $x_t$could be there, while effect of past values (x_{t-i} $where\ i > 1$) are removed. Thus, VAR residuals only contain contemporaneous information about the causal feedback from x to y and vice versa. On the other hand, if the residuals extract from the univariate method - Rehman and Malik (2014) – Modified R recursive residuals are used to determine the causal ordering by using PC algorithms of GTA, it should have more power as compared to residuals extracted from VAR model (Fazal et al., 2021a; Fazal et al., 2021b). So, the study also evaluated the performance of Modified PC algorithm in term of size and power properties to find the correct causal ordering which is the second contribution of the study.

Methodology

Graph Theoretic Approach

The basic design of graph theoretic approach is straightforward. Any structural model is converted into causal graph. In a causal graph, each pair of variables is connected through straight lines having arrowheads representing the direction of causation. It is important to show some notation and terms used in graph theoretic approach which are given below[1]:

Causal Links

Causal relations between pair of variables represented by straight lines are known as links or edges. An edge may represent as follows:

- No edge (A   B)
- Undirected edge: (A—B)
- Unidirectional edge: (A→B) or(A←B)

[1]Tayyaba R. (2009). Causation; Case in Point: Energy Sector

Causal Graph

The graph presenting the causal links between set of variables with their direction is known as causal graph.

Direction of Causation

In a causal graph, an arrowhead shows the direction of causation.

Skeleton of the Graph

Merely representation of variables by graph ignoring the arrow headed direction is the skeleton of the graph.

Path

A path is a sequence of causal link between two variables. It may be directed or undirected i.e. If C is a common cause between A and B and A causes B then CAB is a direct path between C and B.

Causal Sufficiency

If every cause of all variable in a graph is also a variable in that graph, then the graph is causally sufficient.

Principal of Common Cause

If we have three variables X, Y, Z and Y←X→Z then Y and Z would be dependent but conditional on X they would be independent, and then X is called the common cause of Y and Z.

Shielded and Unshielded Collider

If the causal connection between X, Y, Z is shown as X→Z←Y and there is no direct path between X and Y then Z is an unshielded collider between X and Y and if there is a direct link between X and Y then Z is said to be a shielded collider.

Causal Search Algorithms for Testing Causality

PC Algorithm

- Generate data series using data generating process
- Application of VAR model and extract residuals series
- Application of PC algorithms in Graph-Theoretic method, treating VAR residuals as original variables.

Modified PC Algorithm

- Generate data series using data generating process
- Application of Rehman and Malik (2014) test called Modified R and extract recursive residuals series.
- Application of Modified PC algorithms in Graph-Theoretic method, treating modified R residuals as original variables.

Steps Involved in PC and Modified PC Algorithms

PC and Modified PC algorithms have five common steps for calculation of causal ordering (Fazal et al 2021c). The major difference among them is the use of residuals series. In original PC causal algorithms, the residuals series of VAR are to be treated as variables, while in Modified PC, one step is changed to a new version that extracts residuals from the univariate methods, i.e., Rehman and Malik (2014), instead of using VAR model residuals. The steps involved in both approaches PC and Modified PC are given below:

- First it constructs the general structure of graph in which all variables are connected through undirected links.
- It then tests unconditional correlation between any two variables. If they are not unconditionally correlated then eliminate those connections.
- It then **t**ests correlation between each two variables conditional on a third variable. If each pair of variables are conditionally uncorrelated then again eliminate their connections.
- This step is called orientation stage. In the previous step 3 if two variables are correlated conditional on the third variable, the members of pair are unshielded colliders on that path, and arrows from the two variables are oriented toward the third variable.
- In this step, arrows are oriented on the basis of screening relationship. If two variables X and Y are not directly linked but are linked through a third variable Z as $X \rightarrow Z$--- Y, that the first link to the third variable is directed and the other link is undirected. So, then orient the second link as $X \rightarrow Z \rightarrow Y$ because orienting the arrow toward Z shows that Z is unshielded collider and if it is true then this should be revealed in step 4. Thus, the intervening variable is a screen and not an unshielded collider, so the arrow cannot point toward it.

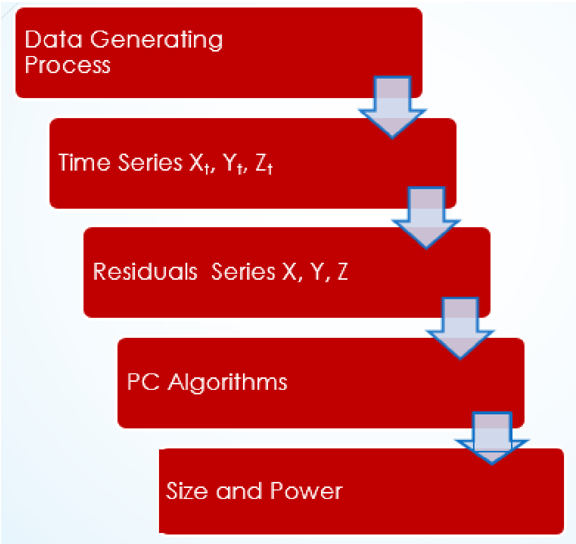
In application Fisher's z statistics is used to test whether the conditional correlations are significantly different from zero.

Testing Procedure for Comparing PC and Modified PC Algorithms

The given below flow chart explains two different methodologies which differ only in residuals series. The methodologies are VAR residuals and Modified R recursive residuals. The steps involved in methodology are explained and summarized in the following flow chart.

Figure 1

Flowchart of the Procedure for Assessing the Size and Power of the PC and Modified PC Algorithms



First, the data series X_t, Y_t and Z_t are generated using the data generating process (DGP). Using the generated three time series, the residual series are extracted by applying the VAR model and the Rehman and Malik (2014) test. These extracted residuals are then referred to the PC and Modified PC algorithms, respectively, and the performance of these procedures is evaluated using size and Power properties.

Simulation Methodology

Data Generating Process

Objective of the study is to evaluate the performance of PC and

Modified PC algorithms by investigating size and power properties, Monte Carlo Simulation is conceived. We have the following data generating process (DGP) in Equation (1).

$$\begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \theta_1 & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_2 & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_3 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix} + \begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix} \quad (1)$$

The above matrix form equation can be written in the following form:

$$X_t = AX_{t-1} + BDt + \varepsilon_t \varepsilon_t \sim N(0, \Omega)$$

Where$A = \begin{bmatrix} \theta_1 & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_2 & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_3 \end{bmatrix}$,$B = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$, $Dt = \begin{bmatrix} 1 \\ t \end{bmatrix}$, $\varepsilon_t = \begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$,$\Omega = \begin{bmatrix} 1 & \rho 1 & \rho 2 \\ \rho 1 & 1 & \rho \\ \rho 2 & \rho & 1 \end{bmatrix}$

Equation (1) is general DGP which generates data in different ways by imposing different restrictions on matrix A and B. The series with drift and trend can be generated by taking B $\neq$ 0. The parameter B is called "nuisance". The causality does not depend on the matrix of parameter B, however the test statistics for coefficient present in "A" which determine causality is heavily dependent on B and incorrect specification of B may create bias. So, to avoid the biasness we have to include this nuisance term.

Results and Discussions

Size Distortion as Measure of Performance

It is well known that powers of econometric tests/procedures are comparable if the size remain same, and so is the case with the two approaches mentioned above. But size cannot be controlled in the PC algorithm of Graph Theoretic Approach because the causality testing through PC algorithms involve multiple decisions in a chain and standardization of size is not possible when many decisions are involved. Usually, when tests are to be compared, the process starts by finding out the critical values with fixed size, say 5%. These critical values are then used to calculate power curves. However, PC algorithms involves multiple testing, therefore 5% critical values for the entire procedures cannot be calculated. Alternatively, we can measure size distortion where the size of entire procedure can be calculated fixing the size each single step at 5%.

The test with minimum size distortion would be the optimal test. The best performance would be considered as of the procedure having minimum size distortion and highest power.

Let alpha be the size of a test/procedure then

$\alpha = P(Reject\ H_0/H_0\ is\ True)$

In our case, the null hypothesis H_0: there is no causality between "x and y" and for calculation of size, the data should be generated such that $H_o$is true. The alternative hypothesis in our case is "H_1: x causes y".

It is also important that size for PC and Modified PC algorithms are calculated from different stationary and non-stationary series with wide range of specification of deterministic part and autoregressive coefficients.

Size Analysis with Non-Stationary Series

In this section, size properties of PC and Modified PC causal algorithms are evaluated. First, three independent autoregressive non-stationary series x_t, y_t and $z_t$are generated using data generating process (DGP) given in equation (1). Non stationary series are generated with different specifications (drift and trend) by imposing various restrictions on matrix B. In DGP, we take A as diagonal matrix which means, we have not put any causal relationship among the three generated series. The generated series are then referred to different testing procedures i.e., VAR and Modified R, and residuals are extracted. Using VAR residuals and Modified R recursive residuals in PC and Modified PC algorithms respectively, we got various magnitude of probability of spurious causal relationship for all possible directions $x \to y,\ y \to z$, and $x \to z$.

Table 1

Probability of Rejection of the Hypothesis of No Causality using PC and Modified PC Algorithms with Non-stationary Series Having No Drift and Trend

	ρ	PC	Modified PC
		VAR Residuals	Modified *R* Residuals
		No Drift and No Trend	
$x \to y/no\ causaliy$	1	0.063	0.481
$y \to z/no\ causaliy$	1	0.079	0.446

	ρ	PC	Modified PC
		VAR Residuals	Modified R Residuals
$x \to z/no\ causaliy$	1	0.074	0.446
Drift only			
$x \to y/no\ causaliy$	1	0.071	0.64
$y \to z/no\ causaliy$	1	0.073	0.64
$x \to z/no\ causaliy$	1	0.075	0.64
Both Drift and Trend			
$x \to y/no\ causaliy$	1	0.072	0.632
$y \to z/no\ causaliy$	1	0.071	0.649
$x \to z/no\ causaliy$	1	0.064	0.649

The results given in Table 1 indicate the probability of rejection of null hypothesis of no causality(true) using PC and Modified PC algorithms. The results obtained from PC algorithm using VAR residuals indicate about 7% on average significant results against 5% nominal size in all three different specifications (i.e., without drift and trend, with drift only and having both drift and trend) as shown in column 2nd of Table 1. This implies there is on the average 2% size distortion which can be regarded as spurious causality because the true DGP does not have causality. Column 3rdindicates the results of Modified PC causal search algorithm, treating Modified R recursive residuals as original variables showing on average 48% significant result for all three different specifications, which means on average size distortion of about 43%. Modified PC algorithm using modified R residuals, probability of incorrect decisions for all possible directions$x \to y,\ y \to z$, and $x \to z$ is much higher than the probability of spurious causality obtained from PC algorithm using VAR residuals.

Size Analysis with Stationary Series

The three independent stationary series have been generated by using data generating process given in equation (1). In case of stationary series, we choose $\theta_{ij} = \begin{cases} \rho & i = j \\ 0 & otherwise \end{cases}$ $where\ \rho < 1$, and impose various restrictions on matrix B, which means stationary series are generated without drift and trend, with drift only and both with drift and trend. Using the generated series, the simulated results are given in Table 2 as under:

Table 2

Probability of Rejection of the hypothesis of No Causality using PC and Modified PC Algorithms in Case of Stationary Series with Different Specifications

	ρ	PC	Modified PC
		VAR Residuals	Modified R Residuals
		No Drift and No Trend	
$x \rightarrow y/no\ causaliy$	0.8	0.063	0.358
$x \rightarrow y/no\ causaliy$	0.6	0.065	0.202
$x \rightarrow y/no\ causaliy$	0.4	0.050	0.123
$x \rightarrow y/no\ causaliy$	0.2	0.055	0.090
$y \rightarrow z/no\ causaliy$	0.8	0.065	0.333
$y \rightarrow z/no\ causaliy$	0.6	0.063	0.194
$y \rightarrow z/no\ causaliy$	0.4	0.084	0.116
$y \rightarrow z/no\ causaliy$	0.2	0.064	0.078
$x \rightarrow z/no\ causaliy$	0.8	0.074	0.333
$x \rightarrow z/no\ causaliy$	0.6	0.073	0.194
$x \rightarrow z/no\ causaliy$	0.4	0.043	0.116
$x \rightarrow z/no\ causaliy$	0.2	0.078	0.078
		Drift only	
$x \rightarrow y/no\ causaliy$	0.8	0.063	0.335
$x \rightarrow y/no\ causaliy$	0.6	0.077	0.185
$x \rightarrow y/no\ causaliy$	0.4	0.072	0.108
$x \rightarrow y/no\ causaliy$	0.2	0.066	0.092
$y \rightarrow x/no\ causaliy$	0.8	0.069	0.335
$y \rightarrow x/no\ causaliy$	0.6	0.070	0.185
$y \rightarrow x/no\ causaliy$	0.4	0.062	0.108
$y \rightarrow x/no\ causaliy$	0.2	0.056	0.092
$x \rightarrow z/no\ causaliy$	0.8	0.066	0.364
$x \rightarrow z/no\ causaliy$	0.6	0.054	0.182
$x \rightarrow z/no\ causaliy$	0.4	0.054	0.109
$x \rightarrow z/no\ causaliy$	0.2	0.077	0.095
		Both Drift and Trend	
$x \rightarrow y/no\ causaliy$	0.8	0.081	0.42
$x \rightarrow y/no\ causaliy$	0.6	0.069	0.217
$x \rightarrow y/no\ causaliy$	0.4	0.082	0.114

	ρ	PC	Modified PC
		VAR Residuals	Modified R Residuals
$x \rightarrow y/no\ causaliy$	0.2	0.062	0.079
$y \rightarrow x/no\ causaliy$	0.8	0.056	0.409
$y \rightarrow x/no\ causaliy$	0.6	0.078	0.208
$y \rightarrow x/no\ causaliy$	0.4	0.069	0.137
$y \rightarrow x/no\ causaliy$	0.2	0.050	0.078
$x \rightarrow z/no\ causaliy$	0.8	0.075	0.409
$x \rightarrow z/no\ causaliy$	0.6	0.067	0.208
$x \rightarrow z/no\ causaliy$	0.4	0.081	0.137
$x \rightarrow z/no\ causaliy$	0.2	0.070	0.070

Table 2 indicates the results of PC and Modified PC algorithms of x causing $y(x \rightarrow y)$,y causing z $(y \rightarrow z)$ and x causing z $(x \rightarrow z)$ at autoregressive coefficients $(\theta_{11}, = \theta_{22} = \theta_{33})$ values 0.8, 0.6, 0.4, and 0.2. The results of PC algorithm using VAR residuals show that the probability of significant results fluctuates around 7% on average for three possible causal directions at nominal size 5% for all three specifications given in column 2nd.It means that there is on average 2% probability of spurious causality in the three possible directions. when we treated modified R recursive residuals as original variables in Modified PC algorithms, the results indicate that when stationary series with root close to unity, the probability of significant results remains high which is about on average 27% as shown in column 3rd of Table 2. But when the autoregressive parameters are close to zero i.e., 0.2, the probability of significant results decrease to7% on average for all three possible directions. This show on the average 2% probability of spurious regression for all three possible directions.

It is concluded from the simulations results that PC algorithm using VAR residuals, stationary series, non-stationarity series, autoregressive coefficient, specifications (drift and trend) do not affect the its size. The results of Modified PC algorithm using modified R recursive residuals indicates that the size depends on value of auto regressive coefficient and its distortion reduces when auto regressive coefficients approach zero. The results indicate that when the series is highly stationary (low memory), then causal algorithm using modified R recursive residuals perform same as VAR residuals in size distortion problem.

Power Analysis of PC and Modified PC Algorithms

Monte Carlo Simulation Design

In this section, the power of the PC and Modified PC algorithms is analyzed. The power of any test is defined as the probability of rejecting null hypothesis when it is false.

$$Power = P(Rejecting\ H_0/H_1\ is\ True)$$

We analyze the power of PC and Modified PC for a variety of situations. We know that the power also depends on several nuisance parameters related to the "deterministic part" as well as "stochastic part". Among the deterministic part are component of drift and trend while among stochastic part, we have the autoregressive coefficient of the three series which also determine the stationarity of the series. This study used sample size of 100 for data generating process under alternative hypothesis to calculate power.

For calculation of power, we use the data generating process used in Equation (1) which is as follows

$$\begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix} + \begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix} \quad (1)$$

where, $\begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho 1 & \rho 2 \\ \rho 1 & 1 & \rho \\ \rho 2 & \rho & 1 \end{bmatrix} \right)$

$$X_t = AX_{t-1} + BDt + \varepsilon_t \varepsilon_t \sim N(0, \Omega)$$

This data generating process will generate causally dependent series when A is non-diagonal matrix. We take $\begin{bmatrix} \theta_{11} & 0 & 0 \\ \theta_{21} & \theta_{22} & 0 \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix} \neq 0$ and ρ1= ρ2 = 0 which implies that there is no contemporaneous correlation among generated series x, y and z. Therefore, x, y and z are serially dependent but have no contemporaneous relationship.

Once the series from DGP in equation (1) are generated, we calculate the residuals of the series through one of the two procedures i.e., VAR model and Modified R test, and are used subsequently in the PC and Modified PC algorithm of graph theoretic approach. Power properties of these approaches will be calculated by finding the probability of each of the two mentioned scenarios i.e., Correct[2] and Omitted[3]. For this analysis, dependent autoregressive stationary and nonstationary time series are being generated with different complications; (with drift and trend), (with drift only) and (with drift and with trend). All the estimated results have been summarized after 10000 times simulations from the data generating process.

Power of PC and Modified PC Algorithms Using Non-Stationary Series

First, we have generated non-stationary series x, y and z on the basis of change in both stochastic and deterministic part using the data generating process given in equation (1). we have imputed double cross terms(θ_{21}andθ_{32}) in matrix A, and power of PC and Modified PC algorithms are calculated. The cross terms establish correlation between x and y and y and z and its value also changes from 0.9, 0.8, 0.6, 0.4 and 0.2 in matrix A which shows that $x \rightarrow y$ and $y \rightarrow z$ respectively. i.e., matrix A $= \begin{bmatrix} \theta_{11} & 0 & 0 \\ \theta_{21} & \theta_{22} & 0 \\ 0 & \theta_{32} & \theta_{33} \end{bmatrix} \neq 0.$

Table 3

Power of PC and Modified PC algorithms using Non-Stationary Series without Drift and Trend

	ρ	PC		Modified PC	
		VAR residuals		MR residuals	
		Correct	Omitted	Correct	Omitted
	$\theta_{11} = \theta_{22} = \theta_{33} = 1$				
$x \rightarrow y/causaliy$	0.9	0.06	0.93	0.65	0.34
$x \rightarrow y/causaliy$	0.8	0.06	0.93	0.63	0.36
$x \rightarrow y/causaliy$	0.6	0.05	0.94	0.60	0.39
$x \rightarrow y/causaliy$	0.4	0.05	0.94	0.62	0.37
$x \rightarrow y/causaliy$	0.2	0.05	0.94	0.60	0.39
$y \rightarrow z/causaliy$	0.9	0.05	0.94	0.57	0.42
$y \rightarrow z/causaliy$	0.8	0.06	0.93	0.55	0.45
$y \rightarrow z/causaliy$	0.6	0.07	0.93	0.53	0.46

[2]The link is present both in data generating process and final results of PC algorithm.
[3]The link is present in data generating process but absent in PC algorithm results

	ρ	PC		Modified PC	
		VAR residuals		MR residuals	
		Correct	Omitted	Correct	Omitted
$y \rightarrow z/causaliy$	0.4	0.07	0.92	0.51	0.48
$y \rightarrow z/causaliy$	0.2	0.05	0.94	0.49	0.50

Table 4

Power of PC and Modified PC Algorithms using Non-Stationary Series with Drift only

	ρ	PC		Modified PC	
		VAR residuals		MR residuals	
		Correct	Omitted	Correct	Omitted
	$\theta_{11}= \theta_{22} = \theta_{33}= 1$				
$x \rightarrow y/causaliy$	0.9	0.07	0.92	0.73	0.26
$x \rightarrow y/causaliy$	0.8	0.06	0.93	0.75	0.24
$x \rightarrow y/causaliy$	0.6	0.06	0.93	0.75	0.24
$x \rightarrow y/causaliy$	0.4	0.07	0.92	0.74	0.25
$x \rightarrow y/causaliy$	0.2	0.05	0.94	0.72	0.27
$y \rightarrow z/causaliy$	0.9	0.07	0.92	0.78	0.21
$y \rightarrow z/causaliy$	0.8	0.05	0.94	0.79	0.20
$y \rightarrow z/causaliy$	0.6	0.06	0.93	0.79	0.20
$y \rightarrow z/causaliy$	0.4	0.06	0.93	0.76	0.23
$y \rightarrow z/causaliy$	0.2	0.06	0.93	0.77	0.22

Table 5

*Power of PC and Modified PC Algorithms using Non-Stationary Series with Drift +*and Trend*

	ρ	PC		Modified PC	
		VAR residuals		MR residuals	
		Correct	Omitted	Correct	Omitted
	$\theta_{11}= \theta_{22} = \theta_{33}= 1$				
$x \rightarrow y/causaliy$	0.9	0.07	0.92	0.73	0.26
$x \rightarrow y/causaliy$	0.8	0.07	0.92	0.75	0.24
$x \rightarrow y/causaliy$	0.6	0.06	0.93	0.74	0.25
$x \rightarrow y/causaliy$	0.4	0.05	0.94	0.73	0.27
$x \rightarrow y/causaliy$	0.2	0.07	0.93	0.76	0.23

	ρ	PC		Modified PC	
		VAR residuals		MR residuals	
		Correct	Omitted	Correct	Omitted
$y \to z/causaliy$	0.9	0.07	0.92	0.76	0.23
$y \to z/causaliy$	0.8	0.05	0.94	0.78	0.21
$y \to z/causaliy$	0.6	0.07	0.92	0.75	0.24
$y \to z/causaliy$	0.4	0.06	0.93	0.77	0.23
$y \to z/causaliy$	0.2	0.06	0.93	0.78	0.21

In the above Table 3 the simulated results of PC and Modified PC algorithms are reported. Each outcome is expressed as a proportion of the number of times it might have occurred. First, non-stationary series, θ_{11}= θ_{22} = θ_{33}= 1 having no drift and trend are generated from the DGP given in equation (1). The generated series are then analyzed in VAR model and Modified R test, residuals series of the said models are extracted which are then used in PC and Modified PC algorithm by treating these residuals as original variables.

Row 1st and row 6th of Table 3, where the coefficients θ_{21} ($x\ cause\ y$)= 0.9 and θ_{32} ($y\ cause\ z$) = 0.9. Using VAR residuals in PC causal search indicate that the probability of rejection of null hypothesis of no causality (which can be regarded as power, since in DGP null is not true) is about on average 7% and 6% respectively. Using Modified R recursive residuals instead of VAR residuals, indicating that the probability of rejection of null of no causality is about 65% for θ_{21} ($x \to y$) and 57% for θ_{32}($y \to z$).

In Table 4, non-stationary series (θ_{11}= θ_{22} = θ_{33}= 1) with drift are generated from the given DGP (1). The cross dependences terms θ_{21} and θ_{32} also changes from 0.9, 0.8, 0.6, 0.4 and 0.2 in matrix A of DGP, which shows that $x \to y$ and $y \to z$ respectively.The simulated results in row 1st and row 6th of Table 4, indicate that using VAR residuals in PC causal search the probability of rejection of null of no causality is about on average 7% and 7% respectively, and this does not change significantly when cross terms θ_{21} and θ_{32} changes from 0.9 to 0.2. Modified PC indicating the probability of rejection of null hypothesis of no causality on average 74% and 78% and this does not vary when θ_{21} and θ_{32} changes from 0.9 to 0.2.

In Table 5, the simulated results of PC and Modified PCs algorithms are given. In this case generated series is non-stationary having both drift and trend. Row 1st and row 6th of Table 5 indicates that the probability of rejection of null of no causality is about 7% for both the cross terms θ_{21} and θ_{32} and displayed the same pattern when the cross terms θ_{21} and θ_{32} changes from 0.9 to 0.2. Using Modified R recursive residuals instead of VAR residuals the results indicating that the probability of rejection of null hypothesis of no causality is about 74% and 76% for both the cross terms θ_{21} and θ_{32} and this does not change significantly when θ_{21} and θ_{32} changes from 0.9 to 0.2.

It is clear from the above simulated results that PC algorithm (using VAR residuals) perform very bad in power properties, when the generated series are nonstationary with different specifications (drift and trend). However, Modified PC (using Modified R recursive residuals) is performing better in all cases. Discussing the other scenario: Omitted error is high when VAR residuals are referred to PC causal search in all three Tables 3, 4 and 5. Modified PC algorithm using modified R recursive residuals perform good with less omission in all cases. Hence, from the above Tables 3, 4 and 5, it is concluded that Modified PC algorithm using modified R recursive residuals works well in case of power than those algorithms using VAR.

Power Comparison of PC and Modified PC Algorithms Using Stationary Series

In previous section the performance of PC and Modified PC algorithms in non-stationary series was analyzed. Tables 6, 7 and 8 summarizes performance of PC and Modified PC causal search algorithms, when the underlying series are stationary. To make the series stationary we put the diagonal entries θ_{11}, θ_{22}, θ_{33} in DGP (1) to be smaller than unity. To create cross dependences, we choose some of the non-diagonal entries to be non-zero. We choose $\theta_{21} > 0$, $\theta_{32} > 0$, and $\theta_{31} > 0$ which make x depending on y, y depending on z and x depending on z respectively. we have imputed causality between (x, y) and (y, z) and then we calculated the power of PC and Modified PC algorithms given in Tables 6, 7 and 8.

Table 6
Power of PC and Modified PC algorithms using Stationary Series without Drift and Trend

	ρ	PC VAR residuals Correct	PC VAR residuals Omitted	Modified PC MR residuals Correct	Modified PC MR residuals Omitted
		$\theta_{11} = \theta_{22} = \theta_{33} = 0.8$			
$x \rightarrow y/causaliy$	0.9	0.06	0.93	0.45	0.54
$x \rightarrow y/causaliy$	0.8	0.06	0.94	0.49	0.50
$x \rightarrow y/causaliy$	0.4	0.07	0.92	0.57	0.42
$x \rightarrow y/causaliy$	0.2	0.06	0.93	0.50	0.49
$y \rightarrow z/causaliy$	0.9	0.06	0.93	0.43	0.56
$y \rightarrow z/causaliy$	0.8	0.07	0.92	0.47	0.52
$y \rightarrow z/causaliy$	0.4	0.06	0.93	0.59	0.40
$y \rightarrow z/causaliy$	0.2	0.05	0.95	0.51	0.48
		$\theta_{11} = \theta_{22} = \theta_{33} = 0.6$			
$x \rightarrow y/causaliy$	0.9	0.06	0.93	0.44	0.55
$x \rightarrow y/causaliy$	0.8	0.06	0.93	0.50	0.49
$x \rightarrow y/causaliy$	0.4	0.06	0.93	0.50	0.49
$x \rightarrow y/causaliy$	0.2	0.06	0.93	0.32	0.67
$y \rightarrow z/causaliy$	0.9	0.06	0.93	0.47	0.52
$y \rightarrow z/causaliy$	0.8	0.06	0.93	0.49	0.50
$y \rightarrow z/causaliy$	0.4	0.06	0.93	0.45	0.54
$y \rightarrow z/causaliy$	0.2	0.07	0.92	0.31	0.69
		$\theta_{11} = \theta_{22} = \theta_{33} = 0.4$			
$x \rightarrow y/causaliy$	0.9	0.05	0.94	0.35	0.64
$x \rightarrow y/causaliy$	0.8	0.05	0.94	0.31	0.68
$x \rightarrow y/causaliy$	0.4	0.07	0.92	0.28	0.71
$x \rightarrow y/causaliy$	0.2	0.08	0.91	0.16	0.83
$y \rightarrow z/causaliy$	0.9	0.05	0.94	0.26	0.73
$y \rightarrow z/causaliy$	0.8	0.06	0.93	0.32	0.67
$y \rightarrow z/causaliy$	0.4	0.07	0.92	0.26	0.73
$y \rightarrow z/causaliy$	0.2	0.06	0.94	0.17	0.82
		$\theta_{11} = \theta_{22} = \theta_{33} = 0.2$			
$x \rightarrow y/causaliy$	0.9	0.06	0.93	0.11	0.88
$x \rightarrow y/causaliy$	0.8	0.06	0.93	0.14	0.85

	ρ	PC		Modified PC	
		VAR residuals		MR residuals	
		Correct	Omitted	Correct	Omitted
$x \rightarrow y/causaliy$	0.4	0.06	0.93	0.12	0.87
$x \rightarrow y/causaliy$	0.2	0.05	0.94	0.09	0.90
$y \rightarrow z/causaliy$	0.9	0.05	0.94	0.12	0.87
$y \rightarrow z/causaliy$	0.8	0.05	0.93	0.15	0.84
$y \rightarrow z/causaliy$	0.4	0.06	0.93	0.12	0.88
$y \rightarrow z/causaliy$	0.2	0.06	0.93	0.08	0.91

Table 7

Power of PC and Modified PC algorithms using Stationary Series with Drift only

	ρ	PC		Modified PC	
		VAR residuals		MR residuals	
		Correct	Omitted	Correct	Omitted
$\theta_{11} = \theta_{22} = \theta_{33} = 0.8$					
$x \rightarrow y/causaliy$	0.9	0.07	0.92	0.42	0.57
$x \rightarrow y/causaliy$	0.8	0.07	0.92	0.39	0.60
$x \rightarrow y/causaliy$	0.4	0.05	0.94	0.44	0.55
$x \rightarrow y/causaliy$	0.2	0.06	0.93	0.50	0.49
$y \rightarrow z/causaliy$	0.9	0.05	0.95	0.42	0.57
$y \rightarrow z/causaliy$	0.8	0.06	0.93	0.42	0.57
$y \rightarrow z/causaliy$	0.4	0.06	0.93	0.42	0.57
$y \rightarrow z/causaliy$	0.2	0.06	0.93	0.50	0.49
$\theta_{11} = \theta_{22} = \theta_{33} = 0.6$					
$x \rightarrow y/causaliy$	0.9	0.06	0.93	0.44	0.55
$x \rightarrow y/causaliy$	0.8	0.08	0.92	0.44	0.56
$x \rightarrow y/causaliy$	0.4	0.05	0.94	0.47	0.52
$x \rightarrow y/causaliy$	0.2	0.06	0.93	0.33	0.66
$y \rightarrow z/causaliy$	0.9	0.06	0.93	0.43	0.56
$y \rightarrow z/causaliy$	0.8	0.07	0.93	0.46	0.53
$y \rightarrow z/causaliy$	0.4	0.05	0.94	0.50	0.49
$y \rightarrow z/causaliy$	0.2	0.06	0.93	0.33	0.66
$\theta_{11} = \theta_{22} = \theta_{33} = 0.4$					
$x \rightarrow y/causaliy$	0.9	0.06	0.93	0.29	0.70
$x \rightarrow y/causaliy$	0.8	0.05	0.94	0.33	0.66

	ρ	PC		Modified PC	
		VAR residuals		MR residuals	
		Correct	Omitted	Correct	Omitted
$x \rightarrow y/causaliy$	0.4	0.06	0.93	0.25	0.74
$x \rightarrow y/causaliy$	0.2	0.05	0.94	0.18	0.81
$y \rightarrow z/causaliy$	0.9	0.06	0.93	0.32	0.67
$y \rightarrow z/causaliy$	0.8	0.07	0.92	0.31	0.68
$y \rightarrow z/causaliy$	0.4	0.07	0.92	0.26	0.73
$y \rightarrow z/causaliy$	0.2	0.05	0.94	0.16	0.84
$\theta_{11}= \theta_{22} = \theta_{33}= 0.2$					
$x \rightarrow y/causaliy$	0.9	0.05	0.94	0.14	0.85
$x \rightarrow y/causaliy$	0.8	0.05	0.94	0.14	0.85
$x \rightarrow y/causaliy$	0.4	0.06	0.93	0.11	0.88
$x \rightarrow y/causaliy$	0.2	0.07	0.92	0.09	0.90
$y \rightarrow z/causaliy$	0.9	0.08	0.91	0.12	0.88
$y \rightarrow z/causaliy$	0.8	0.07	0.92	0.13	0.86
$y \rightarrow z/causaliy$	0.4	0.06	0.93	0.10	0.89
$y \rightarrow z/causaliy$	0.2	0.05	0.94	0.08	0.91

Table 8

Power of PC and Modified PC algorithms using Stationary Series with Drift and Trend

	ρ	PC		Modified PC	
		VAR residuals		MR residuals	
		Correct	Omitted	Correct	Omitted
$\theta_{11}= \theta_{22} = \theta_{33}= 0.8$					
$x \rightarrow y/causaliy$	0.9	0.07	0.92	0.42	0.57
$x \rightarrow y/causaliy$	0.8	0.07	0.92	0.39	0.60
$x \rightarrow y/causaliy$	0.4	0.05	0.94	0.44	0.55
$x \rightarrow y/causaliy$	0.2	0.06	0.93	0.50	0.49
$y \rightarrow z/causaliy$	0.9	0.05	0.95	0.42	0.57
$y \rightarrow z/causaliy$	0.8	0.06	0.93	0.42	0.57
$y \rightarrow z/causaliy$	0.4	0.06	0.93	0.42	0.57
$y \rightarrow z/causaliy$	0.2	0.06	0.93	0.50	0.49
$\theta_{11}= \theta_{22} = \theta_{33}= 0.6$					
$x \rightarrow y/causaliy$	0.9	0.06	0.93	0.44	0.55
$x \rightarrow y/causaliy$	0.8	0.08	0.92	0.44	0.56

	ρ	PC		Modified PC	
		VAR residuals		MR residuals	
		Correct	Omitted	Correct	Omitted
$x \to y/causaliy$	0.4	0.05	0.94	0.47	0.52
$x \to y/causaliy$	0.2	0.06	0.93	0.33	0.66
$y \to z/causaliy$	0.9	0.06	0.93	0.43	0.56
$y \to z/causaliy$	0.8	0.07	0.93	0.46	0.53
$y \to z/causaliy$	0.4	0.05	0.94	0.50	0.49
$y \to z/causaliy$	0.2	0.06	0.93	0.33	0.66
$\theta_{11}= \theta_{22} = \theta_{33}= 0.4$					
$x \to y/causaliy$	0.9	0.06	0.93	0.29	0.70
$x \to y/causaliy$	0.8	0.05	0.94	0.33	0.66
$x \to y/causaliy$	0.4	0.06	0.93	0.25	0.74
$x \to y/causaliy$	0.2	0.05	0.94	0.18	0.81
$y \to z/causaliy$	0.9	0.06	0.93	0.32	0.67
$y \to z/causaliy$	0.8	0.07	0.92	0.31	0.68
$y \to z/causaliy$	0.4	0.07	0.92	0.26	0.73
$y \to z/causaliy$	0.2	0.05	0.94	0.16	0.84
$\theta_{11}= \theta_{22} = \theta_{33}= 0.2$					
$x \to y/causaliy$	0.9	0.05	0.94	0.14	0.85
$x \to y/causaliy$	0.8	0.05	0.94	0.14	0.85
$x \to y/causaliy$	0.4	0.06	0.93	0.11	0.88
$x \to y/causaliy$	0.2	0.07	0.92	0.09	0.90
$y \to z/causaliy$	0.9	0.08	0.91	0.12	0.88
$y \to z/causaliy$	0.8	0.07	0.92	0.13	0.86
$y \to z/causaliy$	0.4	0.06	0.93	0.10	0.89
$y \to z/causaliy$	0.2	0.05	0.94	0.08	0.91

Table 6, 7 and 8 corresponds to stationary series with various complications (drift and trend). The off-diagonal entries $\theta_{21\,,}\, \theta_{32}$changes from 0.9, 0.8, 0.4 and 0.2 in matrix A of DGP which implies that x is causing y and y is causing z. In Table 6.10, row 1st of panel 1st corresponds to series where $\theta_{21}\ and\ \theta_{32}$= 0.9 at autoregressive coefficient (i.e., 0.8). The results show that PC algorithm using VAR residuals and Modified PC algorithms using modified R residuals, the probability of rejection of null of no causality (which can be regarded as power, since in DGP null is not true) is about 6% and 45% respectively (given column **correct**), and this does not change significantly when off-diagonal entries ($\theta_{21}\ and\ \theta_{32}$)

changes from 0.9 to 0.2. The table reveals that the power is best for Modified R residuals while VAR residuals perform bad in case of power. But when the auto regressive coefficient goes to zero, the power of Modified R goes down, as evident from the table, when you move down from panel 1st to panel 8th. Similarly, Table 7 and Table 8 also display the same results as discussed in Table 6. So, the interpretations of these cases are approximately alike as Table 6.

It is concluded from the above discussions that Modified PC algorithm using modified R recursive residuals perform well (with minimum power loss) both in nonstationary and stationary time series with different specifications (drift and trend). But this procedure loss power when the auto regressive coefficient value approach to 0.2 (decrease the diagonal values from 0.8 to 0.2). PC algorithm using VAR residuals present very poor performance (low power).

Conclusion

From the outcome of this study, we can easily deduce that:

- Modified PC algorithm - using modified R recursive residuals were successful in identifying the correct causal links presence in both, DGP and final simulated results with high reliability when the auto regressive coefficient is near to unity. But when the auto regressive coefficient in the data generating process tends towards zero, this procedure fails to perform its function to identify correct causal directions. The PC algorithm - using VAR residuals present very poorly performance in finding the correct causal structure.
- Furthermore, it may be noted that the causal links present in DGP is not removed in the simulated results of Modified PC algorithm - using modified residuals. This means that the omission errors proportion in this procedure are very low as compared to PC algorithm - using VAR residuals.

Conflict of Interest

The authors of the manuscript have no financial or non-financial conflict of interest in the subject matter or materials discussed in this manuscript.

Data Availability Statement

Data associated with this study will be provided by corresponding author upon reasonable request.

Funding Details

No funding has been received for this research.

References

Demiralp, S., & Hoover, K. D. (2003). Searching for the causal structure of a vector autoregression. *Oxford Bulletin of Economics and Statistics*, *65*(5), 745–767.

Demiralp, S., Hoover, K. D., & Perez, S. J. (2008). *A bootstrap method for identifying and evaluating a structural vector autoregression*. Oxford Bulletin of Economics and Statistics.

Fazal, R., Alam, M. S., Hayat, U., & Alam, N. (2021b). Effectiveness of monetary policy: Application of modified Peter and Clark (PC) algorithms under graph-theoretic approach. *Scientific Annals of Economics and Business*, *68*(3), 333–344. https://doi.org/10.47743/saeb-2021-0021

Fazal, R., Bhatti, M. I., & Rehman, A. U. (2022a). Causality analysis: The study of size and power based on riz-PC algorithm of graph-theoretic approach. *Technological Forecasting and Social Change*, *180*, Article e121691. https://doi.org/10.1016/j.techfore.2022.121691

Fazal, R., Rehman, S. A. U., & Bhatti, M. I. (2022b). Graph theoretic approach to expose the energy-induced crisis in Pakistan. *Energy Policy*, *169*, Article e113174. https://doi.org/10.1016/j.enpol.2022.113174

Fazal, R., Rehman, S. A. U., Bhatti, M. I., Rehman, A. U., Arooj, F., & Hayat, U. (2021a). A cross-sectoral investigation of the energy–environment–economy causal nexus in Pakistan: Policy suggestions for improved energy management. *Energies*, *14*(17), Article e5495. https://doi.org/10.3390/en14175495

Fazal, R., Rehman, S. A. U., Rehman, A. U., Bhatti, M. I., & Hussain, A. (2021c). Energy–environment–economy causal nexus in Pakistan: A graph theoretic approach. *Energy*, *214*, Article e118934.

Granger, C. W. J. (1969). Investigating causal relations by econometric models and cross-spectral methods. *Econometrica*, *37*(3), 424–438.

Hicks, J. (1980). *Causality in economics*. Australian National University Press.

Hoover, K. D. (2001). *Causality in macroeconomics*. Cambridge University Press.

Hoover, K. D. (2005). Automatic inference of the contemporaneous causal order of a system of equations. *Econometric Theory*, *21*(1), 69–77.

Lin, J. L. (2008). *Notes on testing causality*. Institute of Economics, Academia Sinica & Department of Economics, National Chengchi University.

Pearl, J. (1993). [Bayesian analysis in expert systems]: Comment: Graphical models, causality and intervention. *Statistical Science*, *8*(3), 266–269.

Rehman, A. U., & Malik, M. I. (2014). The modified R: A robust measure of association for time series. *Electronic Journal of Applied Statistical Analysis*, *7*(1), 1–13. https://doi.org/10.1285/i20705948v7n1p1

Sims, C. A. (1972). Money, income, and causality. *The American Economic Review*, *62*(4), 540–552.

Spirtes, P., Glymour, C. N., Scheines, R., Heckerman, D., Meek, C., Cooper, G., & Richardson, T. (2000). *Causation, prediction, and search*. MIT Press.

Swanson, N. R., & Granger, C. W. J. (1997). Impulse response functions based on a causal approach to residual orthogonalization in vector autoregressions. *Journal of the American Statistical Association*, *92*(437), 357–367. https://doi.org/10.2307/2291475